

Local Constants for Galois Representations - Some Explicit Results

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Given a representation

(1)

$$\rho : Gal(K|F) \rightarrow GL_n(\mathbb{C})$$

of a Galois group of **number fields**, one may form the **augmented** Artin L-series $\Lambda(\chi_\rho, s)$ which is a meromorphic function of a complex variable s depending only on the character χ_ρ of that representation. As a classical result we have the functional equation

(2)

$$\Lambda(\chi_\rho, 1 - s) = W(\chi_\rho) \cdot \Lambda(\overline{\chi_\rho}, s),$$

where $W(\chi_\rho)$ is a complex constant of absolute value 1, the **Artin root number**. (See for instance: An Introduction to the Langlands Program, p.81).

If $\rho = \chi_\rho = \chi$ is 1-dimensional, in his derivation of the functional equation for Hecke L-series, **J. Tate** found a canonical decomposition of $W(\chi)$ into a product over all places ν of F :

$$W(\chi) = \prod_{\nu} W_{\nu}(\chi),$$

where the factors $W_{\nu}(\chi)$ =**local root numbers** depend only on the restriction of χ to the decomposition group $G_{\nu} = Gal(K_w|F_{\nu})$ which comes as the Galois group of an **extension of local fields**.

Langlands (1970) noticed that also the higher-dimensional root numbers $W(\chi_\rho)$ of (2) should have a decomposition into local factors $W_{\nu}(\chi_\rho)$. The existence of these local root numbers has been proved by Langlands himself (in a unpublished preprint) and by Deligne, using global methods.

For a completely local existence proof one has to use a **Brauer map**

$$b_G : R_+(G) \rightarrow R(G), \quad [H, \varphi] \rightarrow Ind_H^G(\varphi),$$

which realizes virtual representations of a (pro)finite group G in terms of 1-dimensional characters for subgroups H and to describe $Ker(b_G)$ in terms of generating relations. Then it has to be verified that Tate's local root numbers $W_{\nu}(\chi)$ for 1-dimensional characters respect these generating relations.

In the talk we derive some explicit formulas for the local root numbers $W_{\nu}(\chi_\rho)$ if ρ is a Heisenberg representation and (following a paper of H.Koch) think on the role these formulas could play in a local existence proof.