Abstract of doctoral thesis entitled
„Estimations of solutions growth rate for neutral type systems”

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The dissertation is devoted to the differential equations with distributed delay, i.e. such equations, for which the change of state depends not only on the present state, but also on all previous states at a certain time interval preceding the current state. Namely, considered are the equations in space $\mathbb{C}^n$ of the following form

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)d\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)d\theta, \qquad (1)$$

where $\det A_{-1} \neq 0$. The basic approach to the study of this class of equations (taken from the work of Burns, Herdman and Stech 1983) is to interpret them as linear equations in a Hilbert space. Equation (1) corresponds to the equation

$$\dot{x}(t) = \mathcal{A}x(t), \quad x(t) \in H \qquad (2)$$

given by unbounded, densely defined operator $\mathcal{A}$, which generates strongly continuous semigroup of operators $\{T(t)\}_{t\geq 0}$.

The aim of the work is to describe the asymptotic behavior of the norm of solutions of equation (2). This task is a continuation of research of Rabah, Sklyar and Rezunenko, who in one of the joint work in 2005 gave a complete spectral analysis of operator $\mathcal{A}$, which occurs in model (2). They received also the necessary and sufficient conditions for asymptotic stability.

One of main results obtained in dissertation is the theorem describing asymptotics of the norm of semigroup $T(t)$ at $t \to +\infty$ (Th. 4.9), where the norm of semigroup is estimated from above and below by functions of time, which depend on the parameters associated with matrix A_{-1}. Those evaluations show in particular that, if the growth bound of the semigroup is zero, then solutions of equation (2) can grow at most with power rate. Although the growth rate of solutions is significantly lower than the growth rate of corresponding semigroups. In such situation, the question arises whether, despite the unboundedness of some of the solutions, there may be a dense set of initial states, for which the corresponding solutions will tend uniformly to zero with power rate. This question relates to the polynomial stability and is one of the topics studied in the work. In general, it is known that for generators of semigroups in Banach or Hilbert spaces the necessary condition for polynomial stability is sufficiently slow approach of generators' spectrum to the imaginary axis. In the dissertation it is shown (Th. 3.8), that the proper location of the spectrum is a sufficient condition for polynomial stability of a certain class of semigroups with a discrete spectrum.

The dissertation also includes some applications of obtained results in the control problem for equation (1). Namely, the rate of growth of the diameter of the zero reachable set for a class of bounded controls is described, and the problem of regular stabilizability is discussed.