Summary

The main result of the thesis is the following:

Theorem 1. *Let $\widehat{\varphi} = \phi_1^{e_1} \times \cdots \times \phi_t^{e_t}$ be a t-module where ϕ_i, $1 \leq i \leq t$ are pairwise non-isogenous Drinfeld modules defined over $\mathcal{O}_K$ and such that* $\mathrm{End}\phi_i = A$. *Let $N_i \subset \phi_i(\mathcal{O}_K)$ be a finitely generated A-submodule of the Mordell-Weil group. Pick $\Lambda \subset N = N_1^{e_1} \times \cdots \times N_t^{e_t}$ to be an A-submodule. Let $d_i = rank(\phi_i) \geq e_i$ for each $1 \leq i \leq t$. Let $P \in N$ and assume that for almost all primes $\mathcal{P}$ of A we have* $\mathrm{red}_{\mathcal{P}}(P) \in \mathrm{red}_{\mathcal{P}}(\Lambda)$. *Then $P \in \Lambda + N_{tor}$.*

A motivation for considering such problem was the analogous theorem by G. Banaszak and P. Krasoń proved in their 2011 paper „On arithmetic in Mordell-Weil groups". They gave a homological sufficient condition for a local to global principle to hold.

The strategy of proof is similar to that in the above mentioned paper:

1. Proof of reduction theorem by using a Kummer theory and an appropriate density theorem;
2. Calculations in Mordell-Weil group with usage of its arithmetic and some facts from the theory of semisimple modules and algebras.

However, there are significant technical problems due to the specific properties of considered t-modules. It was possible to obtain the reduction theorem, because R. Pink proved few years ago the Ribet-Bashmakov Kummer Theory in the context of Drinfeld modules. In our situation the adequate reduction theorem is of the following form:

Theorem 2. *Let L/K be a finite extension. Let $x_{i,j} \in \phi_i(\mathcal{O}_L)$ for $1 \leq j \leq s_i$ be linearly independent elements over A for each $1 \leq i \leq t$. There is an infinite set of prime ideals $\mathcal{W}$ of $\mathcal{O}_L$ such that* $\mathrm{red}_{\mathcal{W}}(x_{i,j}) = 0$ *in $\phi_i^{\mathcal{W}}(\mathcal{O}_L/\mathcal{W})$ for all $1 \leq j \leq s_i$ and $1 \leq i \leq t$.*

In the following part of the thesis we give the counterexample to Theorem 1 with assumption that there exists i, $1 \leq i \leq t$, for which $rank(\phi_i) < e_i$. This is inspired by the 2010 paper of P. Jossen and A. Perucca „A counterexample to the local-global principle of linear dependence for abelian varieties", but their method is generalized.

The last part of dissertation treats about „dynamical local to global principle", formulated by S. Barańczuk. S.Barańczuk gave a set of axioms for the dynamical local to global principle to hold in an algebraic system. We checked these axioms for the t-modules under consideration.

Chapter 1 starts with a review of various local-global problems. In its second part the main results of the thesis are formulated. Chapter 2 has an auxiliary character and contains necessary facts from the theory of semisimple modules and algebras. Chapter 3 is an overview of the basic theory of Drinfeld modules and t-modules. The reduction theorem is proved in chapter 4. Chapter 5 contains the proof of the main result and the counterexample. Chapter 6 is devoted to the dynamical local-global principle.